Majorana and neutrinos

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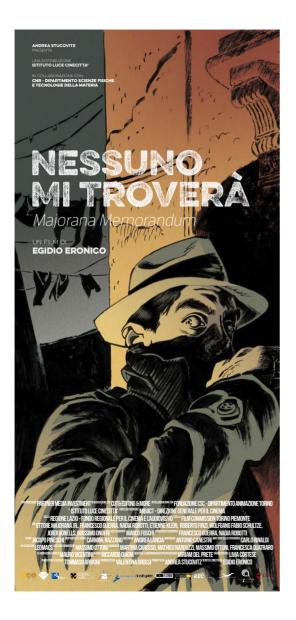
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N. DI MATRICOLA 101-1-7 5906 TESSERA DI RICONOSCIMENTO Il sig a Maiorana Utore Iglio di Fakio Massuno nato a lassuna Man è stato immatricolato studente nella R. Universit di ROMA al ____ anno per gli stuni hor ull. di listo. il giorno Firma dell'Immatricolato never, BOLLI DEGLI ANNI SCOLASTICI DEGLI 82 1923-19

Ettore Majorana (1905-1939) is a famous theoretical physicist, with an intense scientific activity lasting less than a decade. His achievements include notable results in atomic physics, molecular physics, nuclear physics, and elementary particle physics.

He is mostly known for his sudden disappearence at the end of March 1938, which is almost universally still considered as "misterious". This fact helped a lot to influence a quite distorted view of his overall personality. We have dedicated to Ettore Majorana the monograph "Ettore Majorana: aspects of his scientific and accademic activity", edited by the Pisa Scuola Normale Superiore in 2008, and many research papers. Our views are summarized in a recent booklet

Francesco Guerra and Nadia Robotti, La straordinaria vita di Ettore Majorana (The extraordinary life of Ettore Majorana) which accompanies the DVD of the movie "Nessuno mi troverà - Majorana memorandum" (Nobody shall find me), by the movie director **Egidio Eronico**.



The involvement of Majorana with the neutrino is witnessed in his last paper Ettore Majorana, *Teoria simmetrica dell'elettrone e del positrone* (Symmetrical theory of the electron and the positron), Nuovo Cimento **5**, 171-184 (1937).

Here we see a reprint with an handwritten dedication

A S.E. Enrico Fermi con molti cordiali saluti Ettore Majorana "S.E." means "Sua Eccellenza" (His Excellency). Enrico Fermi (1901-1954) as a member of the Accademy of Italy was worth of this title by law. The date on the reprint is important, as we will see, April 1st, 1937. A S. E. Encico Fermi con molti cordiali saluti Ettore Majorana

ETTORE MAJORANA

TEORIA SIMMETRICA DELL'ELETTRONE E DEL POSITRONE

Estratto dal Nuovo Cimento, Anno XIV, N. 4 Aprile 1937



NICOLA ZANICHELLI, EDITORE BOLOGNA 1937-XV

G. MORPURGO

There are two similar reprints with the same dedication (without "S.E." of course): one to Gian Carlo Wick (1909-1992), the other to Giovanni Gentile jr (1906-1942). We do not know whether Giulio Racah (1909-1965) received also a similar gift.

The content of this famous paper is very well known. We will describe the genesis of the paper, and its role for the full professor competition for a chair of theoretical Physics at the University of Palermo. We reconstruct the conceptual itinerary followed by Ettore Majorana on the subject of relativistic quantum wave equations, by resorting to original sources, in particular the research notes kept at the Domus Galilaeana in Pisa.

The main points are:

- his strong criticism against the formulation given by Paul Dirac

- the establishment of the unitary infinite dimensional representations of the Lorentz group

- the discovery of the positron

- the lucid understanding that relativistic quantum wave equations can receive a proper physical interpretation only in the frame of quantum field theory

- the establishment of the symmetric theory for electrons and positrons

- finally the proposal of the Majorana neutrino.

Majorana contributions have great relevance even in the present time, not only for their direct physical contents, as the ongoing research on the Majorana neutrino shows, but also for their methodological strength and originality.

Our treatment here will be very short and schematic. For a more extended version we refer to a forthcoming paper by F.G. and Nadia Robotti "Ettore Majorana on relativistic quantum wave equations", EPJ-H, to appear.

As a matter of fact, the period when Majorana is involved on this subject is quite short (essentially 1932-1934), even though the final paper on Majorana neutrino is published after many years in 1937, on occasion of the participation to the competition for a chair in Theoretical Physics at the University of Palermo.

Dirac equation

It is very well known that Dirac theory originates from a relativistic generalisation of the quantum mechanical Schrödinger equation, leading to Dirac equation.

The Dirac particle is immediately identified with the electron, with the right spin and magnetic momentum properties.

There is a weak point in the theory, but this is at the same time a reason of strength. There are negative energy states. Negative energy states are at the time a real difficulty for the Dirac equation, as they can give rise to many paradoxical phenomena.

It is very well known the way exploited by Dirac in order to overrun the difficulties. In the celebrated picture of the Dirac sea, all energy states are occupied, according to Pauli principle. A hole in the sea can be interpreted as a positive charge particle. In the initial stage, and for a long period, the positive charge hole is recognised as the proton. In order to understand the reasons of Majorana criticism, let us read some sentences in Dirac papers, well surviving for example even in the 1931 french edition of his book "Principles of quantum mechanics" (first english edition May 29, 1930).

In the paper: A Theory of Electrons and Protons, P. A. M. Dirac, Proc. R. Soc. Lond. A 1930 126, published 1 January 1930, one can read In the X-ray case the holes should be counted as things of negative energy, since to make one of them disappear (*i.e.*, to fill it up), one must add to it an ordinary electron of positive energy. Just the contrary holds, however, for the holes in our distribution of negative-energy electrons. These holes will be things of positive energy and will therefore be in this respect like ordinary particles. Further, the motion of one of these holes in an external electromagnetic field will be the same as that of the negative-energy electron that would fill it, and will thus correspond to its possessing a charge + e. We are therefore led to the assumption that the holes in the distribution of negative-energy electrons are the protons. When an electron of positive energy drops into a hole and fills it up, we have an electron and proton disappearing together with emission of radiation.

As a matter of fact, in these first attempts, Dirac is sure to having constructed a symmetrical theory of electrons and protons. Moreover, the whole treatment leads to the idea that the spin 1/2 for the particle is a necessary consequence of quantum mechanics and relativity.

[Further known developments lead to the result that the hole must have the same mass of the electron. It is a kind of anti-electron (Dirac, May 1931), later called positron. From the encounter of the electron and the hole, the electron can be absorbed in the sea. Both disappears, and the energy is recovered in the form of a couple of photons.

On the other hand, a photon with sufficiently high energy, passing near a nucleus, can disappear and transfer an electron of the sea from a negative energy state to a positive one. In this way a couple of real particles electron-hole is created. The nucleus is necessary in order to absorb the excess of momentum. The theory of the Dirac sea has enormous potentialities in the physical interpretation.

The result is a great strength for the Dirac equation: in a sense it "predicts" the existence of the positron.]

Majorana criticism

It is expressed with the typical Majorana sharp style (the Great Inquisitor) in the paper:

E. Majorana, *Teoria relativistica di particelle con momento intrinseco arbitrario*, Nuovo Cimento, **9**, 335-344 (1932).

About the Dirac equation, firstly he says: "poiché questa è applicabile solo a particelle con momento intrinseco s = 1/2, ho cercato equazioni analoghe nella forma a quelle di

Dirac, sebbene alquanto più complicate, le quali permettono la considerazione di particelle con momento intrinseco arbitrario, e in particolare nullo." (Since this equation applies only to particles with intrinsic momentum 1/2, I searched for equations analogous to Dirac one, albeit somewhat more complicated, which allow to consider particles with arbitrary spin, in particular null).

Moreover, he remarks that "l'indeterminazione nel segno dell'energia può essere in realtà superata, usando equazioni del tipo fondamentale, solo se la funzione d'onda ha infinite componenti che *non* si lasciano spezzare in

tensori o spinori finiti." (as a matter of fact the indeterminacy in the sign of the energy can be avoided, by exploiting equations of the basic type, only if the wave function has an infinite number of components, which can not be split in finite tensors or spinors).

Therefore, he arrives to a very important discovery. By using infinite component wave equations, it is possible to recover relativistic invariance in a formulation where there are only positive energy states.

The corresponding Lorentz transformations on the wave function turn out to be unitary. In his 1932 paper, Majorana gives the explicit expression of the generators in the particular case, where some invariant of the representation is null. However he claims that "more general infinite dimensional tensors or spinors can be defined for any value of the invariant". In fact, at the Domus Galilaeana in Pisa, we can find personal research notes, where the explicit expressions of the generators are given in the general case.

In conclusion, in 1932 Ettore Majorana gives

all unitary infinite dimensional representations of the Lorentz group. It is a great mathematical discovery, very important also from the physical point of view. Majorana theory applies to particles with any spin. No negative energy states appear.

His pioneering role is fully recognised in the majestic 1939 paper by Eugene Wigner on Annals of Mathematics, where all representations are constructed through elegant group theory methods. The 1932 paper on Nuovo Cimento is considered by Majorana as a preliminary announcement of his results.

The original handwritten manuscript is at the Domus Galilaeana in Pisa (11 pages). The final wording is reached after extensive and substantial cancellation and rewriting. Clearly the argument is completely new and difficult. In the erased parts there is also mention about the time-reversal invariance, not preserved in the published version. To give a flavor of Majorana style, let us read for example a sentence, carefully written in a first moment and then cancelled, concerning the appearance of negative energy states:

"Si è perfino tentato, non senza ardimento, di attribuire realtà fisica agli stati negativi, come se la natura fosse in imbarazzo per la scelta del segno del radicale $\sqrt{m^2c^4 + c^2p^2}$."

(There has been even the attempt, not devoid of temerity, to give physical reality to negative states, just as if the nature would be embarrassed concerning the choice of the sign in the radical $\sqrt{m^2c^4+c^2p^2}.)$

It is possible to follow in complete detail the progress in the formulation of the paper, by reconstructing the various versions of the paper, as shown in our forthcoming EPJ-H.

Let us see one page with extensive cancellations.

reoria relativistica di particelle con momento intrinseco artitrario. La meccanica quantistica permette, come é noto, ma Trattariane relativisticamente inversionte d' particelle con momento angolare 2 27. Per particelle con mo mento angolare difforente de tapo valore, in particolar millo, non si sono travate sin qui equarioni invarianti e limari nell'energia, come esige lo schema quantistico/ in questa nota mostrero che equariani del la porma richierta porsono/in realta continisi per valori arbitrari del promonto intrinseco, Vinultiplo in Veri o merro di ta. La tropia obbliga a considerare accanto agli Mati aventi significato fisico un rumere severchiante di stati pittizi, non realizzati in natura, e la funcione d'andre ha qui infinite componenti in mayo delle quattro che figurano nelle equario mi di Dirac. Ma il thatto più carattoristico della nostre Teoria é che in esse viene meno la simmetria fra parsato e futuro, fosi che esisterio, per es., stati con massa poritiva, o funnciginaria, que non con massa me gatina, come acquade nella Cearia d' Diracp. Sa presence di Nati perivi di senso pirico è in realta una conseguin za necessaria/della forma lineare importa alle equarioris indipendentemente da un'eventuale indeterminarione nel regno / dell'energia, questo lascia presumere, per unalogia, the anche gli stati negativi delle teoria di Dirac up abbiano prodamento de nelle illigitima conservatione dello schema quantistico siell'ambito della relationitar a aloce mella scella di un tipo di equario we the appare a priori sooretto are arou well illegette no conservince della schema quantistico nell'andi-Contella relatività. Te equacione d'anda, in asserva d' campo, d'una particelle ma teriale deve avere, secondo Dirace, la porma:

According to the Majorana careful strategy of publication, a conclusive paper should be sent to the prestigious german journal Zeitschrift für Physik.

In a January 21 (1933) letter to Ugo Bordoni, high officer in the CNR, supporter of the fellowship to Germany, Majorana, just arrived in Leipzig, writes

"Attendo attualmente alla elaborazione di una teoria relativistica per la descrizione di particelle con momento intrinseco arbitrario

che ho iniziata in Italia e di cui ho dato notizia sommaria nel Nuovo Cimento (in corso di stampa)." (I am here involved in the elaboration of a relativistic theory for particles with arbitrary intrinsic momentum, initiated in Italy, about which I have given concise notice in Il Nuovo Cimento (in press)).

While on the following letter of March 3, we find

"Ho inviato alla Zeitschrift für Physik un articolo sulla teoria dei nuclei. Ho pronto il manoscritto di una nuova teoria relativistica delle particelle elementari e lo invierò alla stessa rivista fra qualche giorno." (I have sent to Zeitschrift für Physik a paper on the theory of nuclei. I have ready the manuscript of a new elementary particle theory, which will be sent to the same journal in few days).

Unfortunately, this paper, fully finished, will never appear. It does not exist among the Majorana material in Pisa.

It is clear that until March 1933 Majorana does not believe in the existence of the positron.

Hence his refusal of the Dirac equation. In a letter to Giovanni Gentile jr (February 1933), Majorana says that nobody in Leipzig believes in the positron. The first experimental evidence, obtained by C. Anderson with a cloud chamber in 1932, can be easily interpreted as electrons going in the opposite direction ("è questa anche l'opinione di [Ernest] Rutherford" (this is also the opinion of Rutherford)).

However, the 1933 experiments by Blackett and Occhialini, exploiting a cloud chamber

controlled by Geiger counters in coincidence, dissolve any doubt. The conviction of the physical existence of the positron is rapidly spreading out. In a (famous) letter to Emilio Segrè (May 22, 1933), Majorana communicates the "season news" that in Leipzig the Dirac theory of negative electrons is gaining credit. Heisenberg is seriously involved.

Lipsia, 22-5-33 Curo Signe, mothe generic per la ten parte Micarione. Novita d' stagione: si de pro alle turia d' Dine digli dettrai positiri. Miran. ting a me ourpa renament. Una delle conse quine più interement à de un quant di sufficient unergia que esser assorbit da un compo d'forra originando una coppia at althoni, I'mo positivo e l'altro negativo. Questo getebbe epiegar in parte l'assortine to moonate di mali prenti; Beck ha fate di calcoli e towate il ginste oraline di grandure. La publittà che qualinoglia, calubo d' Bics non deble enere rive dute, é considerata piceata.

The discovery of the positron given more strength to Dirac scheme, and Majorana now does not feel as urgent the publication of his extended paper on the equations for particles with general spin.

This is a reasonable explanation, based on solid physical reasons, for the missed publication of the paper.

The definite confirmation of the physical existence of the positron is a triumph for Dirac equation. Lord Rutherford, as a good experimental Physicist, shows regret about the fact that the positron has been foreseen theoretically, before being seen experimentally.

"The Nobel Prize in Physics 1933 was awarded jointly to Erwin Schrödinger and Paul Adrien Maurice Dirac "for the discovery of new productive forms of atomic theory"" (Nobel motivation)

["Dirac has set up a wave mechanics which starts from the most general conditions. From

the start he put forward the requirement that

the start he put forward the requirement that the postulate of the relativity theory be fulfilled. Viewed from this general formulation of the problems it appeared that the selfrotation of the electron which had previously come into the theory as an hypothesis stipulated by experimental facts, now appeared as a result of the general theory of Dirac.

Dirac divided the initial wave equation into two simpler ones, each providing solutions independently. It now appeared that one of the solution systems required the existence

of positive electrons having the same mass and charge as the known negative electrons. This initially posed considerable difficulty for Dirac's theory, since positively charged particles were known only in the form of the heavy atom nucleus. This difficulty which at first opposed the theory has now become a brilliant confirmation of its validity. For later on, positive electrons, the positrons, whose existence was stipulated in Dirac's theoretical investigation, have been found by experiment." (from the presentation speech of the Chairman of the Nobel Committee).]

The symmetrical theory of electron and positron

After the delusion of the discovery of the positron, Majorana continues his research on the relativistic formulation of elementary particle theory, in the frame of the new emerging physical frame.

In a 1936 letter to the uncle Quirino he declares that he is working on quantum electrodynamics. Moreover, he gives the title "Quantum Electrodynamics" to his proposed program for a free course in the a.y. 1936-1937. Among the arguments we surprisingly find "The quantisation of the Maxwell-Dirac equations. Study of the relativistic invariance. The positive electron and the symmetry of charges."

While clearly the results were obtained many years before, it is only in 1937 that the fundamental paper appears E. Majorana, *Teoria simmetrica dell'elettrone e del positrone*, Nuovo Cimento, **5**, 171-184 (1937).

Only four pages of the original manuscript survive in the Majorana Archive in Pisa. The text is identical to the published version. Where are the other pages?

Here is one of the surviving pages

L"= i = U = [q+(2, A)] V - i = V = [q+(2, A)] U (31) Dalla variarione del potencial elettromagnetic ri deducino allora la request espressioni per la duristà d' carica o di corrent. $\rho = -\epsilon i \left(u^* V - V^* u \right) = -\epsilon \tilde{\Psi} \Psi - \Psi^* \Psi$ (32) 5= ei(u*av-Vau)= e qay-y aq auch expression differiscone de gull en ditet -such role per costant infinit. Il scantos d'alle su U e V de antreno simuctricament in L', equi stanto whe a un continuents d'regues della carica section pu altrica deve concluder de la concellarison de Jate costoute enfinite. Se U e V athediscono alle relation di anteommentabilità Un (q) Us (q') + Us (q') Un (q) = Sons S(q-q') Vn (q) Vs (q') + Vs (q') Vn (q) = ' Sho 5 (q-q') Un (q) Vo (q') + Vo (q') Vn (q) = 0 aquivalut all'ordinario schema di Jordan - Maijun quando si Jaccia y: U+iV. 3 potencial dettinua queties " e i loro meanert conseget raddiformo in rea all ordinarie relarianis d' commutabilità, fortal is. get Pola q 191 - plai Pola = 1 2 (q-q), essends L'approprieste d' Po = - in (i gi + div A) (33) Px = - 1 En; Py= - 1 Eg; Pz= - 1 Ez &' morgin to and d' The part : H= H'+ H"+ H"'. Il primo termine " si deduce de L' recordo la regoli j'ai export. Il secondo Fernine M'si attim secon. do le regale dariche : H''= { [Po q + (P, A) - L''] da, com i i parte P = (Px, Py, Pz). Quant ad H''', eno mis dedune de l'' intéférentement requendes l'unes o l'alles meted to con den errer dets de l'''é purione Tout delle geouderre di samp material de di quelle elettromagnetiche. Questo prova, d'al-The part, to recessito della posirione (5). S'equasione di continuità è volida secupe purche ria doddifatte inizialment insieme and 1' quaris. we tills divergence der Esang. Sogne die la one

This is a really fascinating paper, devoted to interacting quantum field theory of electronpositron-photon (quantum electrodynamics). The equation of motion are derived through a variational principle for noncommuting fields.

From a conceptual point of view there are two essential ingredients.

First of all it is recognised that relativistic wave equations can have a consistent physical interpretation only in the frame of a quantum field theory. Field equations rule the time evolution not for the quantum wave function, as in the nonrelativistic case of Schrödinger equation, but for quantum field operators, which in the noninteracting case can be split in two pieces, one related to particle creation, and the other related to particle annihilation.

In particular, Dirac equation gets its consistent physical interpretation, where there is no place for the Dirac sea.

Here we notice that in the Amaldi Archive at the Department of Physics there is a recollection by the distinguished theoretical physicist Gian Carlo Wick, who communicates that Majorana was aware of the correct interpretation of the Klein-Gordon equation (spin zero) in the frame of quantum field theory at least since 1931, the year of the Rome nuclear physics conference. According to this recollection Majorana anticipates of many years the 1934 results of Pauli-Weisskopf.

The second fundamental ingredient is given by the well known Majorana representation for Dirac matrices. Dirac and Majorana choices are physically equivalent, but Majorana representation allows to define separately the electronic and the positronic field. Hence a completely symmetrical treatment for the two particles, as shown in the title of the paper.

Majorana representation allows also to introduce a "new description for neutral particles (neutron and neutrino)", where particle and antiparticle coincide. The neutron is obviously excluded (magnetic momentum, and later baryonic number). Therefore we are left with the possibility that the neutrino, already introduced in the 1933-34 papers of Fermi on beta decay, is a Majorana neutrino. This is the subject of intense contemporary research. There are no doubts that it was Enrico Fermi who proposed the name "neutrino" (in Italian "the neutral small one"), as recalled by Wolfgang Pauli in a comment to the talk by Heisenberg at the Solvay meeting in October 1933: "Pour le distinguer des neutrons lourdes, M. Fermi a proposé le nom 'neutrino'."

que leur masse ne peut pas dépasser beaucoup celle de l'électron. Pour les distinguer des neutrons lourds, M. Fermi a proposé le nom « neutrino ». Il est possible que la masse propre des neutrinos soit égale à zéro, de sorte qu'ils devraient se propager avec la vitesse de la lumière, comme les photons. Toutefois leur pouvoir [It should be remarked that Majorana repre-

sentation for the Dirac matrices, is already contained in Dirac, and exploited by him to prove that the complex conjugate of the wave function for a positive energy state is the wave function of a negative energy state.

But Majorana extends these considerations in his quantum field theoretical frame, reaching two important results, the symmetric theory for the electron and the positron, and the possibility for the existence of a neutral particle identical to its own antiparticle.] Majorana neutrino is largely treated in this conference, also in its connection with condensed matter physics, and cosmological problems.

In conclusion, we see that Ettore Majorana is also remarkable for the lucid methodological procedures exploited in his research, where phenomenological considerations, based on physical intuition, are blended with advanced mathematical methods, relying on symmetry and group theoretical methods.

The timing of the paper

Ettore Majorana was not very lucky in his academic career. After the Laurea graduation in 1929, he was not offered any position at the Institute of Physics in via Panisperna, in contrast to all other young "via Panisperna boys'' (Emilio Segr'e, Edoardo Amaldi, Gian Carlo Wick). He earned his "libera docenza" in 1932, a title enabling to give a free course at the university, but was never put in condition to really give a

course. Majorana was surely outside the development planning of the Institute.

However, in 1933 he was given a fellowship by the National Council of Research, and went to Leipzig, at the Heisenberg Institute, to work on the theory of nuclei, where he developed the Majorana exchange forces among proton and neutron.

After the trip abroad Majorana did not publish anything since the last paper in March 1933. In 1936 the rumor spread that the University of Palermo could call for a national competition for a Chair in Theoretical Physics. The last one in Italy was held in 1926 (Enrico Fermi called to Rome).

In order to understand the events, it is necessary to recall the rules. Some University asks for a competion to some Chair. The Minister checks that there is a good cultural reason, and the existence of financial support, and issues an official call on the State Gazette. The candidates have some months to submit their claims, before a definite deadline. Then the appointing committee is nominated by the Minister. After a careful perusal of the presented documents, the Committe choses three winners in a given order. The first in the list is called at the Faculty where the competition was asked, the others in other Faculties, according to the possibilities.

The competition is by titles only, essentially given by the scientific publications. Winners can be easily foreseen.

The official call for the Palermo competition was issued in March 15, 1937. The deadline for candidate application was June 15, 1937.

Since Majorana stopped publications in 1933, he was not a reasonable candidate, as devoid of scientific continuity in his activity.

The other possible and sure candidates were Gian Carlo Wick, Giulio Racah, and Giovanni Gentile jr, here listed in the obvious order, according to the strength of their scientific curriculum. As a matter of fact, Emilio Sagrè, the professor of Experimental Physics in Palermo, had already taken contacts with Wick, the sure first winner, in order to have him in the Faculty.

But Majorana shows a deep and subtle knowledge of academic practice. Just AFTER the official call (March 15) he submits to Nuovo Cimento his paper on the symmetric theory, which is published BEFORE the deadline (June 15), and can be presented to the competition. By a magic timing, Majorana, who can not be a reasonable candidate before March 15, becames automatically the first among the others before the deadline of June 15. In fact, his last paper, added to the previos remarkable ones, makes him the candidate with the best curriculum.

The timing is very strict. To publish the paper before March 15 was dangerous. Since Majorana was the sure first winner in this case, the Minister could be "convinced" to not issue the call, due to some "administrative reason".

Even the "strange" behavior of Majorana in the months before (long hair, living at home, refusing correspondence, ...) did reinforce the idea that he did not have the intention to participate.

At this point, it seems that Majorana would reach his objective to become Professor in native Sicily, as many members of his Family, but it was not so. The powerful academic Authorities could not accept a drastic change in the planning, due to external forces.

The Committee, chaired by Enrico Fermi of course, recognized the outstanding scientific value of Majorana, hesitated to apply the normal rules (comparison between candidates), and suggested the Minister to nominate Majorana as Professor in some University of the Kingdom, outside of the frame of the competition. Therefore, Majorana was in any case eliminated from the competition, and sent to exile at the University of Naples. He disappeared after few months in March 1938.